BEATING HEART OR
THE DESIGNER AND MATHS

How to create a *heart* and how to organise its *beating* to make a special *Valentine*.

Try it for yourself. Suggest different ideas.

If necessary, use some of the ideas listed below. We have used the dynamic software *GeoGebra*.

1. *Beating heart* by using a photo

- We can draw a *heart* with the *pencil* tool or we can use a picture, which we can insert in a *GeoGebra* file. In either case, we will have an image.
- Create a slider (parameter) **a**.
- Create a segment **AB** with length **a.**
- Fix two of the edges of the image at the end of the segment **AB.**
- If the slider is amended from 4 to 5 with a step of 1, this means that in the animation mode we will see sequentially two positions on the image: at **a=4** and **a=5**. We're going to experiment with the values of the slider, especially with the speed. The appropriate value of the speed is important for modelling the heartbeat.

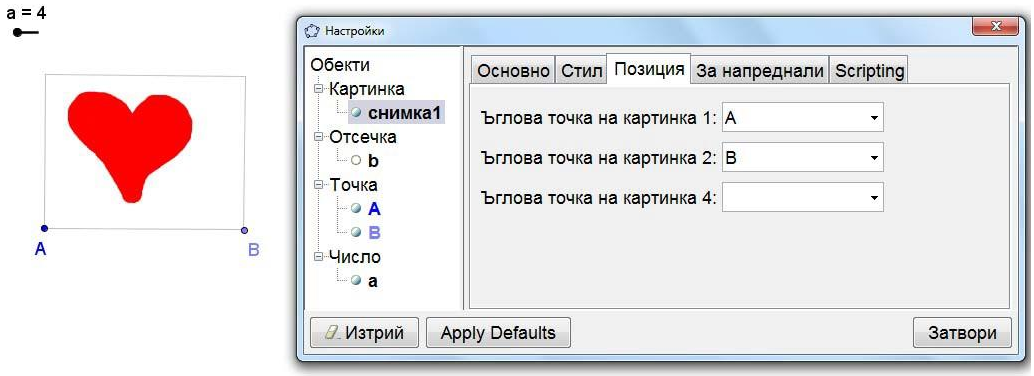


Fig. 1 http://www.math.bas.bg/omi/cabinet/content/bg/html/d25051.html

- We can also create a slider for the speed to change it as appropriate. Then in the speed field we record not a particular number but the name of the new parameter.

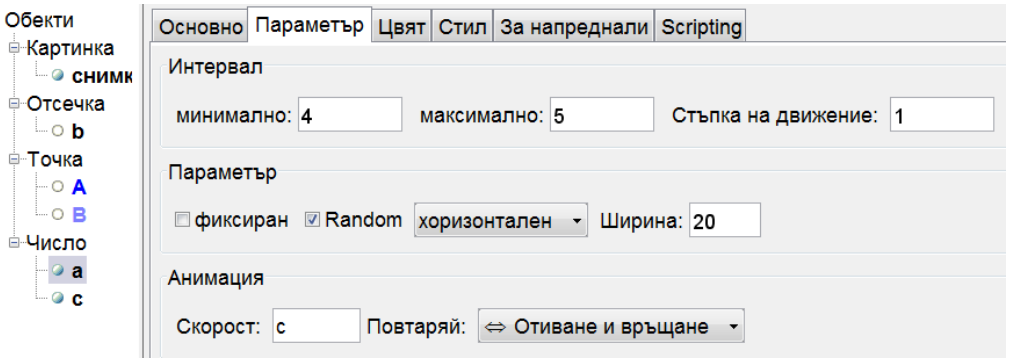


Fig. 2 http://www.math.bas.bg/omi/cabinet/content/bg/html/d25052.html

- We hide the unnecessary objects or their names.
- We insert a suitable text that will make our Valentine unique.

2. *Beating heart* by using two or more pictures

Each of the pictures appears at a specific value of the scroll bar – in this case, the parameter **c** is amended in the interval from 1 to 2 with a step of 1. One of the pictures shows at condition **c=1**, and the other at **c=2**. With points **A**, **B**, **C** and **D**, manage the size and the location of the images.

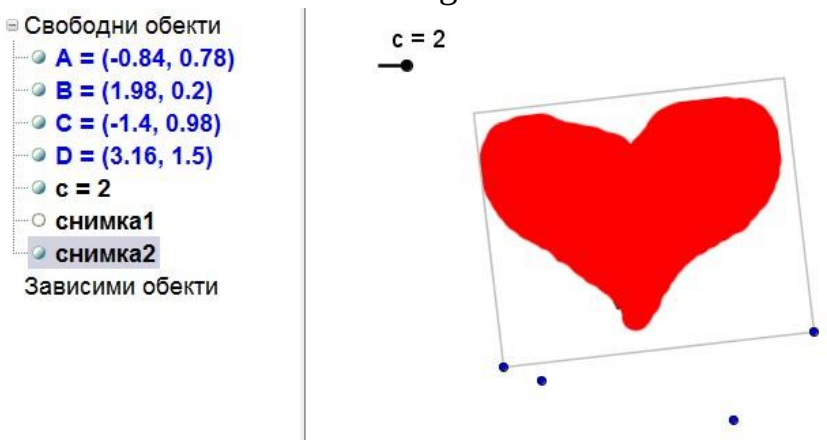


Fig. 3

3. *Beating heart* by using photo and homothety

One picture is sufficient to create the *beating heart* using the homothety. For this purpose, we must give a centre and coefficient. For the coefficient we will use parameter **c**. We will explore the different intervals of the parameter **c**, as well as the various steps to achieve the desired result. For the homothety we can use the taskbar with button or we can enter from the command line:

Dilate[picture1, c, O]

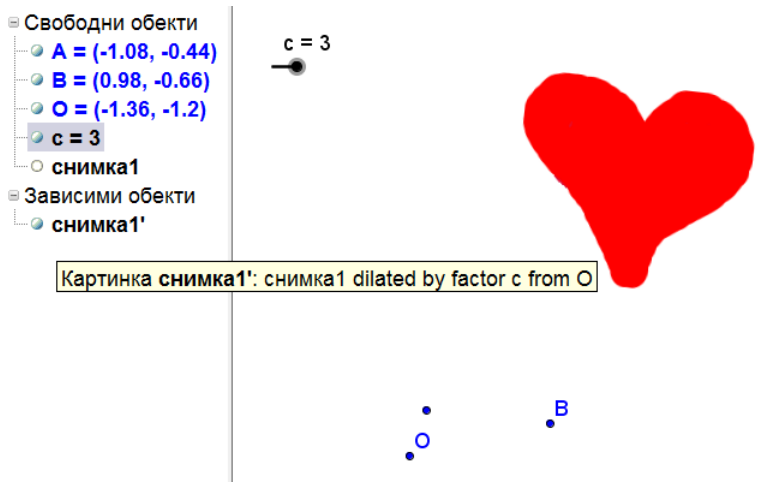


Fig. 4 http://www.math.bas.bg/omi/cabinet/content/bg/html/d25053.html

4. *Beating heart* by using a square and two semi-circles

We have solved the task of finding the surface of a figure composed of a square and two half-circles, which has the shape of a heart. Let's use it for the *beating heart*! Try it for yourself. If you find it hard, use the sequence:

- Create a slider (parameter) **a**.
- Create a segment **AB** with length **a.**
- Construct a square with side **AB** (use the button for the regular polygon). Create the centres **E** and **F** on two adjacent sides of the square.
- Create two semi-circles, **FCD** and **EBC (**use the button for the semi-circle).
- Paint the square and the two semi-circles in red and adjust the transparency to 100%.
- Set the parameter **a** in animation mode.

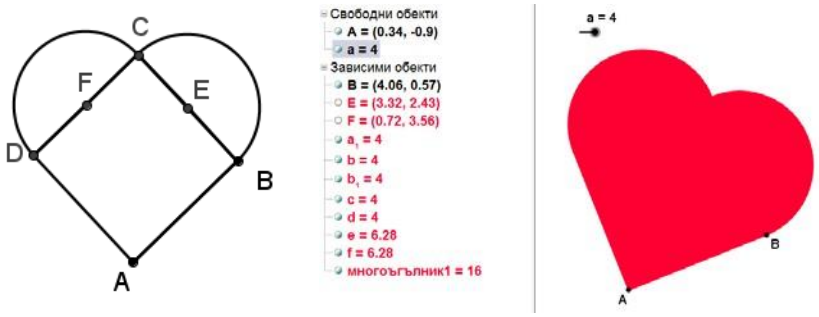


Fig. 5 http://www.math.bas.bg/omi/cabinet/content/bg/html/d25054.html

Let's check what happens if instead of using a square we use a triangle (equilateral, maybe isosceles), or a deltoid, or a semi-circle (consider how to deploy parts of the circles).

Here are some uses:

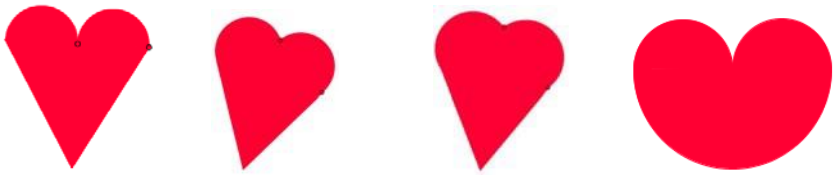


Fig. 6
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25055.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25056.html

Below you will see a created square and circles with centres in the corners and radii equal to half of its side. Can you see in this composition a *heart*? I've used also a white colour to get a *pulsating heart*.

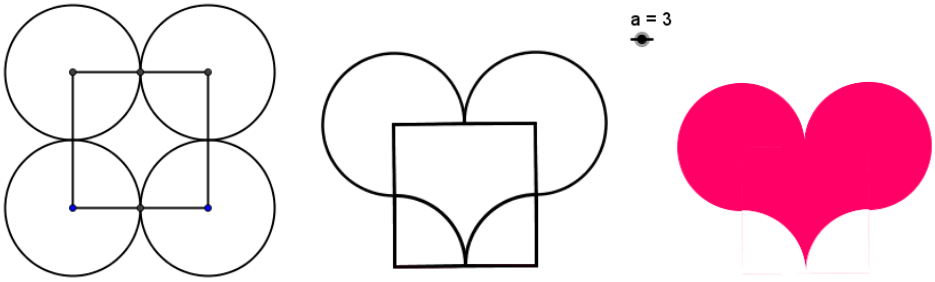


Fig. 7 http://www.math.bas.bg/omi/cabinet/content/bg/html/d25057.html

5. *Beating heart* by using an ellipse

- Construct a slider (parameter, angle) α.
- Construct an ellipse (e.g. through two focuses and its point).
- Construct its images via rotation with a centre – the centre of the ellipse and corners α and $-\alpha$.

Depending on the ellipse and the limits of variation of angle α we will get figures that more or less resemble a *heart*. We have limited α in the interval [30°; 60°].

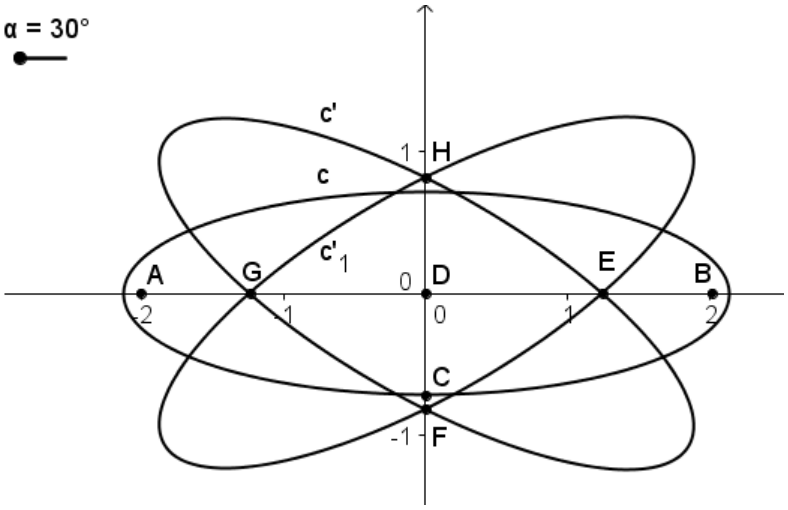


Fig. 8 http://www.math.bas.bg/omi/cabinet/content/bg/html/d25058.html

We can get a *heart* from parts of the ellipses.

- Construct:

Arc[c', H, F]
Arc[c'_1, F, H]

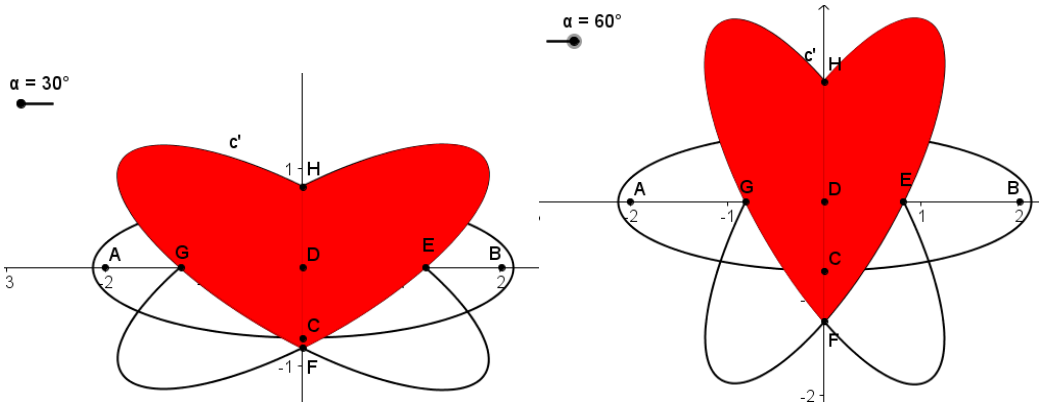


Fig. 9 http://www.math.bas.bg/omi/docs/valentinka/val_24.html

or

Arc[c', H, E]
Arc[c'_1, E, H]

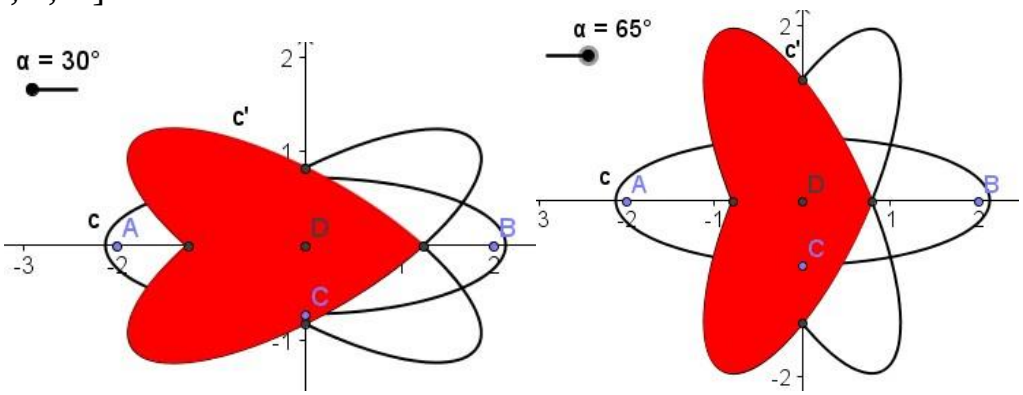
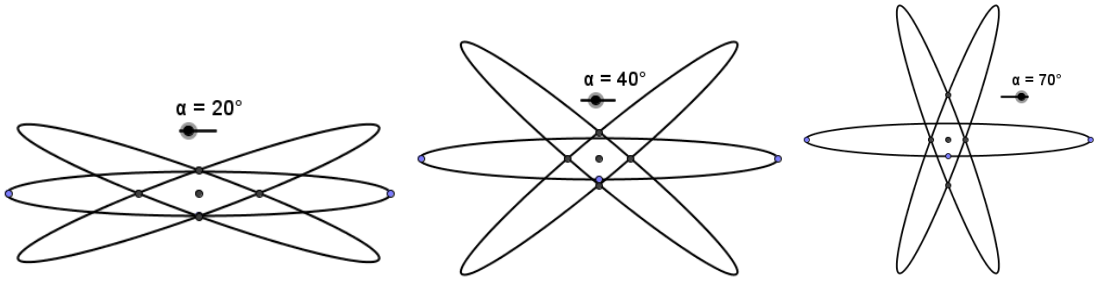


Fig. 10 http://www.math.bas.bg/omi/docs/valentinka/val_25.html

Here are a few more personal cases (do not turn off any of them, the choice of the parameters depends on the purpose):



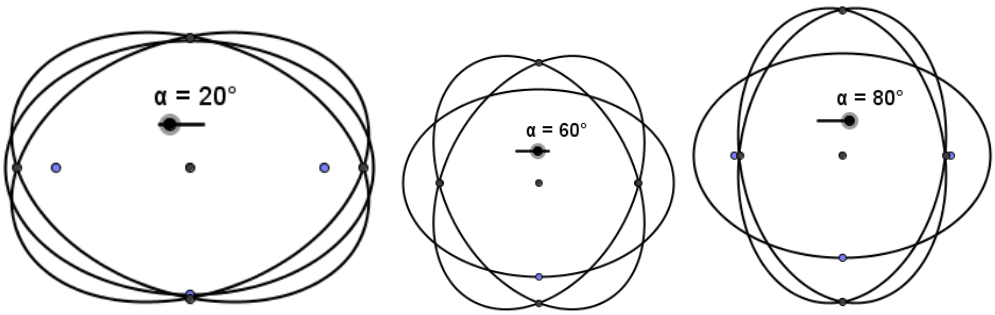


Fig. 11
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25059.html

6. *Beating heart* by using graphs of functions

To model the *beating heart* we can also use a part of a parabola, part of a sine wave...

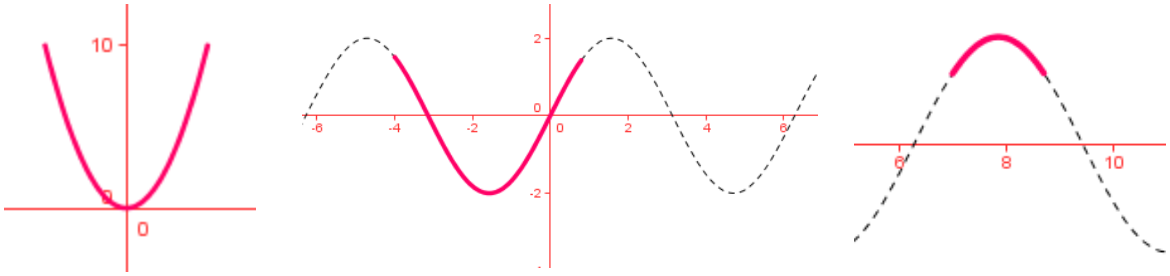


Fig. 12 http://www.math.bas.bg/omi/docs/valentinka/val_31.html

Here are some graphs of functions that looks like parts of a *heart*. Parameters A provide the heartbeat.
f(x) = abs(a x) +
sqrt(b² - x²) g(x) =
abs(a x) - sqrt(b² - x²)

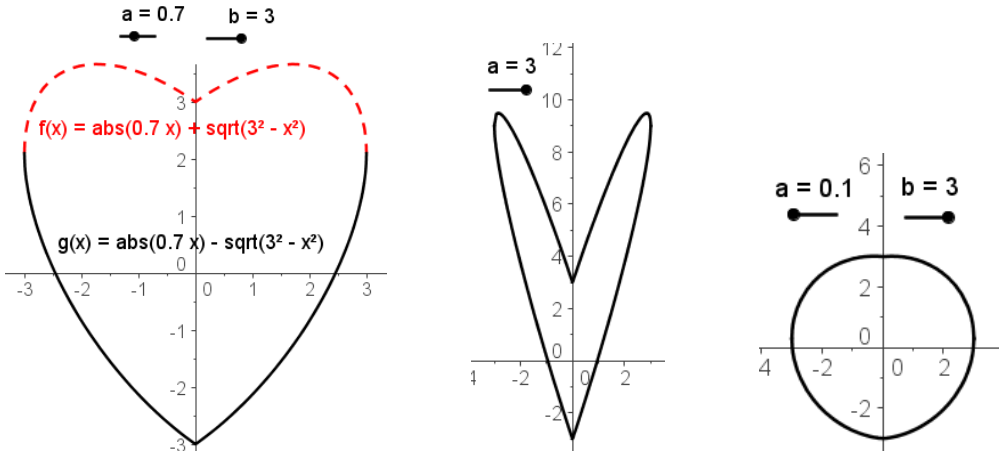


Fig. 13

The beating can be created by **a**, by **b,** or by **a** and **b** together. Of course, we can choose the specific value of one parameter and change the interval and the step of the other. To get the desired model, we experiment. And after a few attempts we can prognosticate more successfully.

7. ***Beating heart* by using curves**

Create a scroll bar (parameter) **a** and define a *pulsating heart* like a curve, for example:

Curve[a $\sin(t)^3$, a $(\cos(t) - \cos(t)^4)$, t, 0, 6.28]

Curve[a $\sin(t)^3$, a $(\cos(t) - \cos(t)^6)$, t, 0, 6.28]

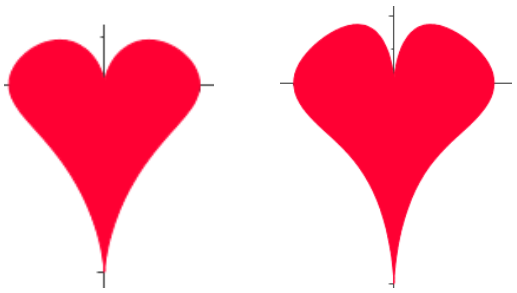


Fig. 14
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25060.html

We will use symmetry to divide the *heart* into two halves, which come close to obtaining one object. One possibility is by dividing the interval and including one more parameter **b** for the distance between the two parts:

Curve[b + a $\sin(t)^3$, a $(\cos(t) - \cos(t)^4)$, t, 0, 3.14]

Curve[-b + a $\sin(t)^3$, a $(\cos(t) - \cos(t)^4)$, t, π, 6.28]

Fig. 15
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25066.html

It is not difficult to depict two or more *beating hearts*, to change the shape:

Curve[2 a $\sin(t)^3$, 2.5 a $(\cos(t) - \cos(t)^6)$, t, 0, 6.28]

Curve[5 + a $\sin(t)^3$, 3 + a $(\cos(t) - \cos(t)^6)$, t, 0,

6.28] $(x^2 + y^2 - 1)^3 = a\, x^2\, y^3$
$(x^2 + y^2 - 3)^3 = a\, x^2\, y^3$

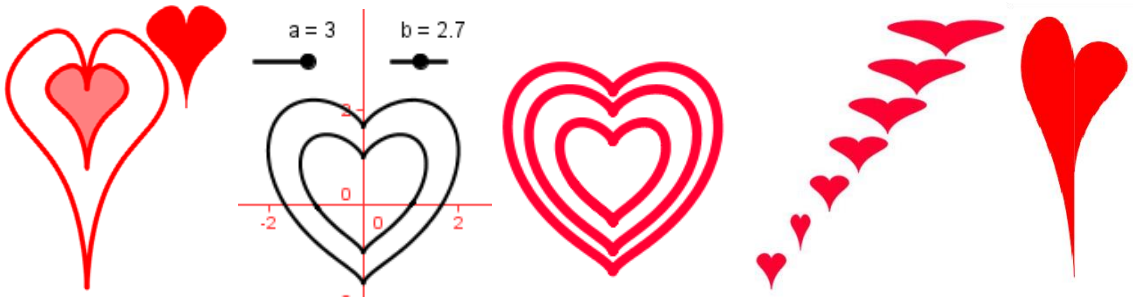


Fig. 16
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25066.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25067.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25069.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25069.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25070.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25071.html

We come to the curve, which not coincidentally is called **cardioid**. This is how we can get a *beating heart* by using it:

$$(x^2 + y^2 - 2a\, x)^2 = 4a^2\, (x^2 + y^2)$$

or

Curve[(a - a sin(k)) cos(k), (a - a sin(k)) sin(k), k, 0, 6.28]

or

Curve[a (1 - cos(k)) sin(k), a (1 - cos(k)) cos(k), k, 0, 6.28]

or

Curve[a (1 - sin(k)) sin(k), a (1 - sin(k)) cos(k), k, 0, 6.28]

or... we always get a cardioid.

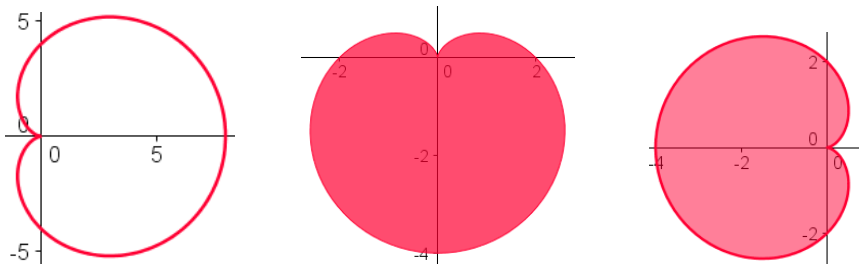


Fig. 17
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25062.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25072.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25073.html
http://www.math.bas.bg/omi/cabinet/content/bg/html/d25074.html

Our goal was to make a Valentine with a *beating heart*. If you set the goal of learning more about the cardioid, even more interesting is the

appearance as a geometrical locus of points.

Notes:

- To work with dynamic files, you must have **java** installed.
- **From** http://www.math.bas.bg/omi/cabinet/index.php?appletid=25**, you can download most of the above dynamic files also as GeoGebra files.**
- To be able to use them, you must have **GeoGebra** installed

http://www.geogebra.org/cms/bg/
http://www.math.bas.bg/omi/cabinet/index.php?appletid=25

References

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http://wiki.geogebra.org/en/Locus_Command (03.09.2013)
http://www.math.bas.bg/omi/Fibonacci/archive.htm (03.09.2013)
http://www.math.bas.bg/omi/docs/Sleda/index.html (03.09.2013)
http://www.math.bas.bg/omi/docs/valentinka/index.html (03.09.2013)